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A Small Cosmological Constant and Backreaction of Non-Finetuned Parameters

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Abstract

We include the backreaction on the warped geometry induced by non-finetuned parameters in a two domain-wall set-up to obtain an exponentially small Cosmological Constant Λ_4. The mechanism to suppress the Cosmological Constant involves one classical fine-tuning as compared to an infinity of finetunings at the quantum level in standard D=4 field theory.

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Starting with heterotic M-theory [1], [2] as the prototypical fundamental brane world scenario where Grand Unification (GUT) becomes also a unification with higher dimensional gravity a lot of activity has been invested to explore this and other brane worlds. One of the model-independent phenomenological successes of heterotic M-theory is that with the input of the GUT scale M_{GUT} and the value of the GUT gauge coupling α_{GUT} it predicts a lower bound on the effective four-dimensional Newton's Constant G_N which coincides strikingly with its measured value [3]. The geometries of these brane worlds are typically described by warped geometries which allowed to solve the hierarchy problem in a novel way which makes essential use of the warped extra dimensions [4]. It is thus natural to investigate the usefulness of warped geometries also for the largest hierarchy problem, the cosmological constant problem [5]. While it was proposed in [6] that warped geometries could be used to explain a vanishing cosmological constant, it was proposed in [7] that they might lead to a mechanism for obtaining a small cosmological constant of order $\Lambda_4 \simeq (\text{meV})^4$. While these are mechanisms which assume a field-theory framework to deal with the issue of the cosmological constant one should keep in mind that in string-theory there are examples of three-dimensional vacua with negative and zero cosmological constants which are connected with each other through T-Duality and therefore obscure the precise low-energy meaning of the vacuum energy [8]. There might also be a completely new understanding of the vacuum energy if M-theory turns out to be a theory of only a finite [9] but huge amount of discrete chain-like degrees of freedom as suggested in [10] based on microscopic black hole entropy derivations. The non-local chains and the associated discreteness of spacetime should shed some new light on how quantum field theory has to be modified in the UV.

We will in this paper restrict ourselves to the traditional field-theory framework and explore further the mechanism for obtaining a small cosmological constant proposed in [7]. The mechanism used a five-dimensional set-up consisting of two four-dimensional positive-tension $T > 0$ domain-walls (there is no need for either the bulk or the walls to be supersymmetric) separated by a distance $2l = M_{GUT}^{-1}$ (M_{GUT} = Grand Unification scale) along the fifth noncompact dimension. Together with bulk gravity and a non-positive bulk cosmological constant $\Lambda(x^5) \leq 0$ the set-up is described by the action

$$S = -\int d^5x \left(\sqrt{-G}\left[M^3 R(G) + \Lambda(x^5)\right] + \sqrt{-g^{(4)}}T\left[\delta(x^5+l) + \delta(x^5-l)\right]\right) . \qquad (1)$$

Neither of the walls is conceived as hidden but instead they are both thought of as being the origin of the Standard Model fields. E.g. by a string-embedding of the set-up and the realisation of the domain-walls as two stacks of D3-branes, one can think of the Standard-

Model gauge group SU(3) as arising from one stack and the SU(2) $\times$ U(1) from the other [7]. For finetuned parameters this set-up leads to a warped geometry containing a flat 4-dimensional spacetime section

$$ds^2 = e^{-A(x^5)}\eta_{\mu\nu}dx^\mu dx^\nu + (dx^5)^2 \;\; ; \;\; \mu,\nu = 1,\ldots,4$$
$$A(x^5) = \frac{k}{2}\left(|x^5+l| + |x^5-l|\right) \;\; , \;\; k = \sqrt{-\Lambda_e/3M^3} \tag{2}$$

with the bulk cosmological constant $\Lambda(x^5)$ and wall-tension T given by

$$\Lambda(x^5) = \begin{cases} \Lambda_e & , \; |x^5| > l \\ \Lambda_e/4 & , \; |x^5| = l \\ 0 & , \; |x^5| < l \end{cases} \;\; , \qquad T = \sqrt{-3M^3\Lambda_e} \; . \tag{3}$$

such that the effective four-dimensional Cosmological Constant Λ_4 vanishes. In this paper, we want to determine the full backreaction of non-finetuned parameters on the warped geometry and demonstrate that with one fine-tuning the resulting effective Λ_4 comes out exponentially suppressed thus freeing the effective four-dimensional theory from the need to correct the cosmological constant order by order in perturbation theory. The suppression-length will turn out to be given by the distance between both walls and will have to be chosen by the inverse of the GUT scale as explained in detail in [7]. To this aim, we have to determine the resulting 5-dimensional geometry for general non-positive $\Lambda \leq 0$ and positive $T > 0$.

Let us start with a D-dimensional warped geometry

$$ds^2 = G_{MN}dx^M dx^N = f(x^D)g_{\mu\nu}(x^\rho, x^D)dx^\mu dx^\nu + (dx^D)^2 \; , \tag{4}$$

where $\mu,\nu,\rho = 1,\ldots,D-1$ and $f(x^D)$ denotes the warp-factor. While the warp-factor in this Ansatz is supposed to be a differentiable function of x^D, the dependence of the 4-dimensional metric $g_{\mu\nu}(x^\rho, x^D)$ on x^D is assumed to be piecewise constant with possible jumps only occuring at the position of the two walls. The freedom to allow for such jumps at the walls comes from the observation that one should expect the walls to have some finite thickness. Most conservatively this is estimated to be of the order of the D-dimensional Planck-length. In a fundamental brane-world theory like heterotic M-theory [1] into which the present two wall set-up could be embedded, the Planck-length is actually larger than naively expected – it is of the same order as the inverse of the Grand Unification scale [11] and indeed is believed to correspond to the width of the walls (orbifold fixed planes) [2]. Therefore, the width seems large enough not to neglect *a priori* variations in the $(D-1)$-dimensional curvature over the width. If the variation occurs

rapid enough then we can model the situation in the approximation of infinitesimal width by a jump in the $(D-1)$-dimensional geometry. Indeed, it has been shown in [12] that fundamental brane world theories very similar to [2] which originate from M-theory and use M9 branes for the walls can only be constructed if one allows for a stepwise constant $(D-1)$-dimensional curvature paired with a stepwise constant D-dimensional cosmological constant with jumps at the brane positions (where $D=11$ for the case of M-theory). Similarly, when we turn to $D=5$ we will later distinguish between a 4-dimensional cosmological constant $\Lambda_4(x^5)$ which is piecewise constant and valid on 4-dimensional sections of the 5-dimensional spacetime and Λ_4 which appears after integrating over x^5 in the effective 4-dimensional action and is of course x^5 independent. As long as the 4-dimensional observer cannot resolve the fifth dimension the parameters of the effective 4-dimensional theory are obtained by integrating out the fifth dimension.

The induced metric on a $(D-1)$-dimensional section defined by $x^D = \text{const}$, will be denoted by $g^{(D-1)}_{\mu\nu}(x^\rho, x^D) = f(x^D) g_{\mu\nu}(x^\rho, x^D)$. Our aim is to solve the Einstein equation piecewise in x^D (such that $g_{\mu\nu}(x^\rho, x^D)$ is constant with respect to x^D on every such piece and x^D) to first determine the lower-dimensional $\Lambda_4(x^5)$ for the case $D=5$. Therefore, we decompose the D-dimensional Ricci-tensor R_{MN} into its μ and D components

$$\begin{aligned} R_{\mu\nu}(G) &= R_{\mu\nu}(g) + \frac{1}{4} g_{\mu\nu} \left(2f'' + (D-3) f \left[(\ln f)' \right]^2 \right) \\ R_{\mu D}(G) &= 0 \\ R_{DD}(G) &= \frac{1}{4}(D-1) \left(2\frac{f''}{f} - \left[(\ln f)' \right]^2 \right) \ . \end{aligned} \tag{5}$$

This allows to decompose the D-dimensional Einstein-tensor $E_{MN}(G) = R_{MN} - \frac{1}{2} R(G) G_{MN}$ as

$$\begin{aligned} E_{\mu\nu}(G) &= E_{\mu\nu}(g) + g_{\mu\nu} \frac{(D-2)}{2} \left[\left(1 - \frac{(D-1)}{4} \right) f \left[(\ln f)' \right]^2 - f'' \right] \\ E_{\mu D}(G) &= 0 \\ E_{DD}(G) &= -\frac{1}{2f} R(g) - \frac{(D-1)(D-2)}{8} \left[(\ln f)' \right]^2 \ . \end{aligned} \tag{6}$$

Let us now restrict ourselves to the $D=5$ case, where the expressions simplify to

$$\begin{aligned} E_{\mu\nu}(G) &= E_{\mu\nu}(g) - \frac{3}{2} g_{\mu\nu} f'' \\ E_{\mu 5}(G) &= 0 \\ E_{55}(G) &= -\frac{1}{2f} R(g) - \frac{3}{2} \left[(\ln f)' \right]^2 \ . \end{aligned} \tag{7}$$

For the action (1) specifying the set-up, the gravitational sources consist of a non-positive bulk cosmological constant $\Lambda(x^5) \leq 0$ and walls with tension T placed at $x^5 = l$ and $x^5 = -l$, which amounts to the following energy-momentum tensor

$$T_{MN} = -\Lambda(x^5) G_{MN} - T \left[\delta(x^5 + l) + \delta(x^5 - l)\right] g^{(4)}_{\mu\nu} \delta^\mu_M \delta^\nu_N \ . \tag{8}$$

Decomposing the 5-dimensional Einstein-equation, $E_{MN}(G) = -T_{MN}/(2M^3)$, with the help of (7) into its μ and 5 components, we receive from the $\mu\nu$ part the 4-dimensional Einstein-equation

$$E_{\mu\nu}(g) = \left[\frac{3}{2} f'' + \frac{f}{2M^3} \left[\Lambda(x^5) + T\delta(x^5 + l) + T\delta(x^5 - l)\right]\right] g_{\mu\nu} \ . \tag{9}$$

From the 55 part follows an expression for the 4-dimensional curvature scalar

$$R(g) = -f \left[3 \left[(\ln f)'\right]^2 + \frac{\Lambda(x^5)}{M^3}\right] \ , \tag{10}$$

whereas the $\mu 5$ part is satisfied trivially.

Contraction of $E_{\mu\nu}(g)$ with $g^{\mu\nu}$ gives $E^\mu{}_\mu(g) = \frac{3-D}{2} R(g) \to -R(g)$ and therefore leads to the following consistency equation among (9) and (10)

$$2\frac{f''}{f} - \left[(\ln f)'\right]^2 = -\frac{1}{3M^3} \left[\Lambda(x^5) + 2T\delta(x^5 + l) + 2T\delta(x^5 - l)\right] \ . \tag{11}$$

It is evident that the right-hand-sides of (9) and (10) must be piecewise constant with respect to x^5, since both left-hand-sides are at least piecewise independent of x^5. This is a consequence of the simple warp-factor Ansatz. It means that the 4-dimensional sections Σ_4, defined by $x^5 = const$, must be spacetimes of constant curvature. For $R(g) < 0$ we have de Sitter (dS_4) and for $R(g) > 0$ Anti-de Sitter (AdS_4) spacetime. Since this already determines the solution to the Einstein equation up to a scalar quantity – the curvature – the equations (9),(10),(11) become linear dependent and it suffices to solve only two of them.

When we foliate the 5-dimensional spacetime into sections Σ_4, we see that the Einstein-equations (9),(10) also follow from the 4-dimensional action on Σ_4

$$S_{D=4}(x^5) = -\int_{\Sigma_4} d^4x \sqrt{-g} \left(M^2_{\text{eff}} R(g) + \Lambda_4(x^5)\right) \tag{12}$$

if we make the following identifications[2]

$$\frac{3}{2}f'' + \frac{f}{2M^3}\left[\Lambda(x^5) + T\delta(x^5+l) + T\delta(x^5-l)\right] = \frac{\Lambda_4(x^5)}{2M_{\rm eff}^2} \tag{13}$$

$$-f\left[3\left[(\ln f)'\right]^2 + \frac{\Lambda(x^5)}{M^3}\right] = -\frac{2\Lambda_4(x^5)}{M_{\rm eff}^2} \ . \tag{14}$$

Notice that the dependence of $S_{D=4}(x^5)$ and of the cosmological constant $\Lambda_4(x^5)$ on sections with respect to x^5 is a piecewise constancy. Here $M_{\rm eff}$ is the effective Planck-scale, as obtained by integrating the 5-dimensional action (1) over x^5

$$M_{\rm eff}^2 = M^3 \int dx^5 f(x^5) \ . \tag{15}$$

The Einstein equations (9),(10) now become replaced by (13),(14).

To recognize the relation between the piecewise constant cosmological constant $\Lambda_4(x^5)$ on sections Σ_4 and the final effective Λ_4 obtained by integrating out the fifth dimension of (1), we note that Λ_4 is given by [7]

$$\Lambda_4 = \int dx^5 f^2\left(M^3\left[[(\ln f)']^2 + 4\frac{f''}{f}\right] + \left[\Lambda(x^5) + T\delta(x^5+l) + T\delta(x^5-l)\right]\right) \ . \tag{16}$$

Using (13) for the second term in square brackets, we obtain the simple and expected relationship

$$\Lambda_4 = f'f|_{x_L^5}^{x_R^5} + \langle\Lambda_4(x^5)\rangle \ , \tag{17}$$

where x_R^5, x_L^5 denote the right and left boundary of the x^5 integration region and the mean is weighted with the profile of the warp-factor

$$\langle\Lambda_4(x^5)\rangle \equiv \frac{\int dx^5 f \Lambda_4(x^5)}{\int dx^5 f} \ . \tag{18}$$

Since we will see that the total derivative contribution $f'f|_{x_L^5}^{x_R^5}$ will vanish in our case of interest, we learn that the 4-dimensional effective action $S_{D=4}$ is related to the sectionwise action by taking the mean, $S_{D=4} = \langle S_{D=4}(x^5)\rangle$.

Since only two of the equations (11),(13),(14) are independent, it is most convenient to choose (11) to determine the warp-factor in terms of the fundamental "input" parameters

[2] The 4-dimensional sections exhibit

$$E_{\mu\nu}(g) = \frac{\Lambda_4(x^5)}{2M_{\rm eff}^2} g_{\mu\nu} \ , \qquad R(g) = -2\frac{\Lambda_4(x^5)}{M_{\rm eff}^2} \ ,$$

with $\mathrm{dS}_4: \ R(g) < 0, \Lambda_4(x^5) > 0$ and $\mathrm{AdS}_4: \ R(g) > 0, \Lambda_4(x^5) < 0$.

$\Lambda(x^5), M$ and T. In a further step, we will then obtain $\Lambda_4(x^5)$ from (14). Expressing the warp-factor through $f = e^{-A(x^5)}$ and denoting $Y(x^5) = A'(x^5)$, we can write (11) as

$$-2Y' + Y^2 + \frac{\Lambda(x^5)}{3M^3} = -\frac{2T}{3M^3}\left[\delta(x^5+l) + \delta(x^5-l)\right] \ , \tag{19}$$

With the signature-function defined by $\mathrm{sign}(x) = -1$ if $x \leq 0$ and $\mathrm{sign}(x) = 1$ if $x > 0$, the solution to this differential equation is given by

$$Y(x^5) = -\frac{k}{2}\left(\mathrm{sign}(x^5+l) + \mathrm{sign}(x^5-l)\right)\coth\left(\frac{k}{4}\left[|x^5+l| + |x^5-l| - 2a\right]\right) \tag{20}$$

together with the following $\Lambda(x^5)$ profile with arbitrary but non-positive constant $\Lambda_e \leq 0$

$$\Lambda(x^5) = \begin{cases} \Lambda_e & , \ |x^5| > l \\ \Lambda_e/4 \leq 0 & , \ |x^5| = l \\ 0 & , \ |x^5| < l \end{cases} \tag{21}$$

and the wall-tension

$$\frac{T}{3M^3} = k\coth\left(\frac{k}{2}(a-l)\right) \ . \tag{22}$$

Here, as in the introduction, $k = \sqrt{-\Lambda_e/3M^3}$ and a is an integration constant. The last relation which determines a through the bulk cosmological constant Λ_e and the wall-tension T has been gained by satisfying the boundary conditions at the wall-locations, which are encoded in the δ-function terms in (19). A matching of the δ-function terms arising from Y' with those proportional to T leads to (22). The symmetry of the set-up – caused by the equality of both wall-tensions – forces the bulk cosmological constant between them to be zero. A non-vanishing value could be obtained by introducing an asymmetry of the set-up through unequal wall-tensions which we will however not do here. A further integration of Y yields the warp-function

$$A(x^5) = -2\ln\left|\sinh\left(\frac{k}{4}\left[|x^5+l| + |x^5-l| - 2a\right]\right)\right| + b \ , \tag{23}$$

where b is a second integration constant. Note, that the above solution is valid for the parameter-range $T \geq 3M^3k$ as can be easily recognized from (22). If $T < 3M^3k$, we have to substitute a "tanh" for the "coth" appearing in (20) and (22), while (21) remains the same. This amounts to a change from "sinh" to "cosh" in (23) Since we assume a positive wall-tension $T > 0$, the integration constant a is constrained through (22) over the whole parameter-region, $T > 0$, $\Lambda_e \leq 0$, by the lower bound $a > l$.

We have two free integration constants a and b. We will see however below that both are related with each other by the imposition that for the situation with finetuned parameters we should get back a solution with flat 4-dimensional sections. Therefore we will end up with just one free integration constant. This remaining free parameter will be fixed in such a way that the finetuning limit exactly reproduces (2) without any further constant added to $A(x^5)$. This constitutes one fine-tuning at the classical level as $A(x^5)$ is determined by the Einstein equations only up to an additive constant.

The explicit solution shows that the warp-factor $f = e^{-A(x^5)}$ vanishes at $x^5 = \pm a$. If $Q < 0$ (which will turn out to be the AdS_4 case, whereas the dS_4 case is free of singularities) this gives rise to a singular 5-dimensional curvature at these points

$$\lim_{x^5 \to \pm a} R(G) \to \frac{24\Theta(-Q)}{(|x^5| - a)^2} \ , \qquad Q = \frac{T - 3M^3 k}{T + 3M^3 k} \ , \tag{24}$$

where the Heaviside step-function is defined by $\Theta(x) = 0, x < 0$ and $\Theta(x) = 1, x > 0$. Due to the vanishing of the warp-factor at these points we expect a tremendous red-shift in signals originating there. Indeed, let us conceive a wave signal emitted with frequency ν_e at $x^5 = \pm a$. Then that wave will be observed in the interior region $x^5 \in (-a, a)$ with frequency ν_o given by

$$\frac{\nu_o}{\nu_e} = \sqrt{\frac{G_{11}(x^5 = \pm a)}{G_{11}(|x^5| < a)}} = 0 \ , \tag{25}$$

due to the vanishing of the warp-factor at $x^5 = \pm a$. Hence, an infinite redshift makes it impossible for the region $|x^5| \geq a$ to communicate to our world (at least via electromagnetic radiation). Therefore, we should restrict the x^5 integration region to the causally connected interval $x^5 \in (-a, a)$.

Since recently there has been a discussion in the literature [13],[14],[15] about which singularities are permissible and which have better to be avoided, it is interesting to see the verdict on our singularities in the case of $Q < 0$. In [13] it has been argued that in a gravitational system exhibiting a 4-dimensional flat solution together with bulk scalars, only those singularities are allowed, which leave the scalar potential bounded from above. In our case, where we do not have any scalars, the role of the scalar potential is played by the bulk cosmological constant Λ_e (together with the tension T at the wall-positions), which is clearly bounded from above. If the criterion of [13] generalizes to the case where the 4-dimensional metric deviates slightly (since in the end Λ_4 turns out to be exponentially small) from the flat case, we would conclude that the singularities encountered above for $Q < 0$ are of the permissible type.

Furthermore, in [15] a consistency condition has been derived which should hold for the effective cosmological constant obtained by integration over the causally connected x^5-region. We will now demonstrate that this consistency condition is a simple consequence of (13),(14) and the expression (16), which defines Λ_4. Starting with (16) and employing (13),(14) to eliminate the derivatives $[(\ln f)']^2$ and f'', (16) becomes

$$\Lambda_4 = 2\langle\Lambda_4\rangle - \frac{1}{3}\int_{-a}^{a} dx^5 f^2 \left(2\Lambda(x^5) + T\delta(x^5+l) + T\delta(x^5-l)\right) \ . \tag{26}$$

Noticing that $f'f(x^5=\pm a)=0$, we use (17) to obtain

$$\begin{aligned} \Lambda_4 &= \frac{1}{3}\int_{-a}^{a} dx^5 f^2 \left(2\Lambda(x^5) + T\delta(x^5+l) + T\delta(x^5-l)\right) \\ &= -\frac{1}{3}\int_{-a}^{a} dx^5 f^2 \left({T_1}^1 + {T_5}^5\right) \ , \end{aligned} \tag{27}$$

which is nothing but the consistency condition of [15]. Since our solution has been derived from (11),(14) which are equivalent to (13),(14) and we will furthermore only require (16) to obtain Λ_4, we conclude that the consistency condition (27) of [15] should be satisfied for our solution.

After this short intermezzo on singularities, let us proceed by inverting (22), to express a explicitly through the input values T and Λ_e

$$a = -\frac{1}{k}\ln|Q| + l \ , \tag{28}$$

which is valid for both $T \geq 3M^3k$ and $T < 3M^3k$. This shows how the parameters T, M, Λ_e influence the width of the x^5 domain.

In order to determine $\Lambda_4(x^5)$, note that to obey the Einstein equations, we have to fulfill (14). This equation can be used to derive the following expression for $\Lambda_4(x^5)$

$$\Lambda_4(x^5) = \pm\frac{3}{2}e^{-b}M_{\text{eff}}^2 \begin{cases} k^2 & , \ |x^5| > l \\ k^2/4 & , \ x^5 = \pm l \\ 0 & , \ |x^5| < l \end{cases} \tag{29}$$

where the plus-sign applies to the case $T \geq 3M^3k$, whereas the minus-sign applies to the complementary case in which $T < 3M^3k$. Since we do not want to use $\Lambda_4(x^5)$ as an input to determine b, but rather focus on the opposite, we are looking for an additional constraint, which allows for a determination of the constant b. This extra constraint comes from considering a smooth transition to the flat solution (2) with $\Lambda_4 = 0$. As can be seen

from (3), we reach the flat limit by sending $T \to 3M^3k$. Via (28) this limit corresponds to sending the constant $a \to \infty$. Thus we see, that the integration region $x^5 \in (-a, a)$ extends over the whole real line in this limit and the warp-function (23) becomes

$$A(x^5) \to \frac{k}{2}\left(|x^5+l|+|x^5-l|\right)+2\ln 2 - ka + b\ . \tag{30}$$

Thus, to guarantee a smooth transition to the flat solution (2), we have to identify the integration constants a and b as follows

$$b = -2\ln 2 + ka\ . \tag{31}$$

Notice that here we implicitly used the mentioned finetuning as $A(x^5)$ in (2) is only determined by the D=5 Einstein equations up to an additive constant which we have set to zero. Thus, together with (28) and (29) we obtain the following expression for $\Lambda_4(x^5)$

$$\Lambda_4(x^5) = 6e^{-kl}QM_{\text{eff}}^2 \begin{cases} k^2 & , |x^5| > l \\ k^2/4 & , x^5 = \pm l \\ 0 & , |x^5| < l \end{cases} \tag{32}$$

which is valid for both parameter-regions $T \geq 3M^3k$ and $T < 3M^3k$.

Finally, to obtain the effective four-dimensional Λ_4, we have to take the mean of $\Lambda_4(x^5)$. Again using that $f'f(x^5 = \pm a) = 0$, we employ (17) and arrive at

$$\Lambda_4 = \frac{\int_{-a}^{a} dx^5 e^{-A(x^5)}\Lambda_4(x^5)}{\int_{-a}^{a} dx^5 e^{-A(x^5)}} = 12e^{-2kl}M^3kQF(|Q|)\ , \tag{33}$$

where we defined $F(|Q|) = 1 - |Q|^2 + 2|Q|\ln|Q|$. In addition we obtain the following effective Planck-scale

$$M_{\text{eff}}^2 = M^3\int_{-a}^{a} dx^5 e^{-A(x^5)} = 2e^{-kl}M^3\left(l(1-|Q|)^2 + \frac{F(|Q|)}{k}\right)\ . \tag{34}$$

There is an exponential-factor occuring in Λ_4 which is the square of the one occuring in M_{eff}^2. At the classical level (classical in the bulk of the five-dimensional spacetime – the field-theories on the walls are however considered quantum mechanically!) an overall constant e^{-kl} multiplying the whole effective 4-dimensional action

$$\begin{aligned} S_{D=4} &= -\int d^4x\sqrt{-g}(M_{\text{eff}}^2R(g) + \Lambda_4) \\ &= -e^{-kl}\int d^4x\sqrt{-g}(\tilde{M}_{\text{eff}}^2R(g) + \tilde{\Lambda}_4) \\ &= -e^{-kl}\tilde{M}_{\text{eff}}^2\int d^4x\sqrt{-g}(R(g) + \lambda_4) \end{aligned} \tag{35}$$

is immaterial – it simply drops out of the field equation[3]. Therefore, we can neglect the overall factor e^{-kl}. The physically observable cosmological constant – invariant under any overall rescaling – is given by $\lambda_4 = \Lambda_4/M_{\text{eff}}^2 = \tilde{\Lambda}_4/\tilde{M}_{\text{eff}}^2$. With (33) and (34) we thus obtain our final result

$$\lambda_4 = e^{-kl}\left(\frac{6k^2QF(|Q|)}{kl(1-|Q|)^2+F(|Q|)}\right) . \tag{36}$$

Some comments about this formula are in order. First, the physical range of the parameter Q lies between $0 \le |Q| \le 1$, where we presuppose a non-negative wall-tension $T > 0$. The lower bound corresponds to the finetuned flat $\Lambda_4 = 0$ limit, while the upper bound is reached for vanishing bulk cosmological constant $\Lambda_e = 0$. Over that region we have $1 \ge F(|Q| < 1) > 0$ and $F(1) = 0$. Hence, we see that starting with some given values for $\Lambda_e \le 0, M, T > 0$ we obtain a positive or negative λ_4 depending on the sign of Q. For $T > \sqrt{-3M^3\Lambda_e}$ the 4-dimensional spacetime will be dS_4, whereas for $T < \sqrt{-3M^3\Lambda_e}$ it will be AdS_4. Furthermore, we recognise a smooth connection to the case with flat 4-dimensional Minkowski spacetime for finetuned parameters $T = \sqrt{-3M^3\Lambda_e} \Leftrightarrow Q = 0$. Second, there is no need for a finetuning of the fundamental parameters to receive a small λ_4. By adapting the distance $2l$ between both walls, one arrives at a huge enough suppression through the exponential factor such that the observed value could be accounted for. Moreover, thanks to the exponential suppression this does not amount to an extremely large hierarchy between the fundamental scale M and the separation-scale $1/2l$.

Hence at the price of one classical finetuning (the additive integration constant for $A(x^5)$ had been set to zero) one is able to bring the generically Planck-sized contribution to λ_4 coming from the three fundamental parameters M, T, Λ_e down to a small value which for $2l = 1/M_{GUT}$ agrees with the experimental observation that $\lambda_4 \simeq (\text{meV})^4/M_{Pl}^2$ [7]. It is important to realise that these three parameters contain all the quantum corrections coming from matter fields (including the Standard Model ones) which are located on the walls. Therefore one does not require any more an infinite finetuning order by order in loop corrections as in the case of a four-dimensional field theory coupled to four-dimensional gravity to match the Cosmological Constant with its observed value. Of course one of the interesting problems which remains is the stabilisation of the inter-wall distance. Without gauge fields in the bulk, mechanisms like the one proposed in [17] are not applicable and it would probably be most satisfying to embed the five-dimensional brane-world first into

[3]Actually, by starting with a more general Ansatz for the five-dimensional metric in which $(dx^5)^2$ gets replaced by $e^{B(x^5)}(dx^5)^2$ avoids this overall multiplicative constant at all [16].

string- or M-theory and then to study the forces between the branes directly in M-theory [18] and to incorporate generic non-perturbative M-theory effects like open membrane instantons for its stabilisation [19]. Alternatively one might try to use a Goldberger-Wise like mechanism [20] for the stabilization directly in five dimensions.

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